Ch122a
Fall term 1997
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$\boxed{\text{Lecture 17}}$

November 10, 1997
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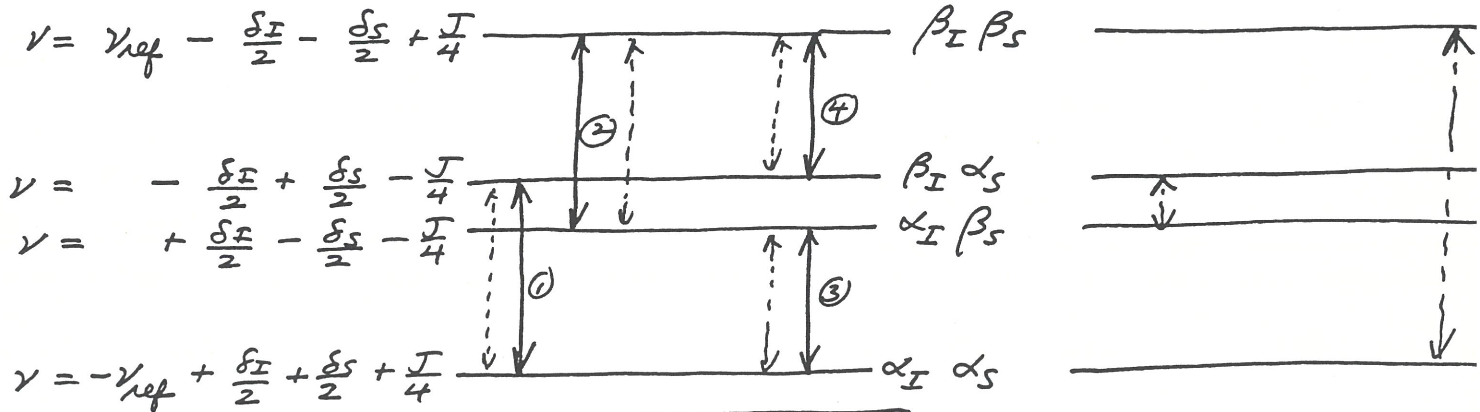
- <u>More on Coherences</u>

Consider an AX spin system made up of an <u>I</u> and a <u>S</u> spin each of spin ½.

<u>Energy level diagram</u>



$\boxed{\text{Energies in units of } h}$

$\boxed{\text{Single-quantum transitions } (\updownarrow)}$

<u>Observable</u>
1. $\nu_{ref} - \delta_I - \frac{J}{2}$
2. $\nu_{ref} - \delta_I + \frac{J}{2}$
3. $\nu_{ref} - \delta_S - \frac{J}{2}$
4. $\nu_{ref} - \delta_S + \frac{J}{2}$

$\boxed{\text{Coherences } (\updownarrow)}$

Operators that lead to connectivities between states & hence <u>coherences</u>

Single-quantum coherences		zero-quantum coherences	double quantum coherences
I_x	S_x	$I_x S_y$	$I_y S$
I_y	S_y	$I_y S_x$	$I_x S_y$
$I_x S_z$	$I_z S_x$		$I^+ S$
$I_y S_z$	$I_z S_y$		$I^- S$

<u>Wavefunctions</u> (time-dependent)

$$\phi_4 = \beta_I \beta_S \cdot \left(e^{-i2\pi(\nu_{ref} - \frac{\delta_I}{2} - \frac{\delta_S}{2} + \frac{J}{4})t}\right) \cdot e^{i\phi_{\beta\beta}}$$

$$\phi_3 = \beta_I \alpha_S \cdot \left(e^{-i2\pi(-\frac{\delta_I}{2} + \frac{\delta_S}{2} - \frac{J}{4})t}\right) \cdot e^{i\phi_{\beta\alpha}}$$

$$\phi_2 = \alpha_I \beta_S \cdot \left(e^{-i2\pi(+\frac{\delta_I}{2} - \frac{\delta_S}{2} - \frac{J}{4})t}\right) \cdot e^{i\phi_{\alpha\beta}}$$

$$\phi_1 = \alpha_I \alpha_S \left(e^{-i2\pi(-\nu_{ref} + \frac{\delta_I}{2} + \frac{\delta_S}{2} + \frac{J}{4})t}\right) \cdot e^{i\phi_{\alpha\alpha}}$$

"phases"

Consider one of the operators that connect a pair of these states say ϕ_1 & ϕ_4.

Then Connectivity given by

$$\int \phi_4^* \, \underset{\sim}{O} \, \phi_3 \, d\tau$$

$$= \int \beta_I^* \beta_S^* \, \underset{\sim}{O} \, \alpha_I \alpha_S \, d\tau \cdot \left[e^{i2\pi(\nu_{ref} - \frac{\delta_I}{2} - \frac{\delta_S}{2} + \frac{J}{4})t} \cdot e^{-i2\pi(-\nu_{ref} + \frac{\delta_I}{2} + \frac{\delta_S}{2} + \frac{J}{4})t} \right]$$

$$\cdot \, e^{-i\phi_{\beta\beta}} e^{-i\phi_{\alpha\alpha}}$$

$$= \int \beta_I^* \beta_S^* \, \underset{\sim}{O} \, \alpha_I \alpha_S \, d\tau \cdot \left[e^{i2\pi(2\nu_{ref} - \delta_I - \delta_S)t} \right] e^{-i\phi_{\beta\beta}} e^{(i\phi}$$

at $t=0$, when all isochromats are in-phase (driven by applied H_1)

$$e^{i2\pi(2\nu_{ref} - \delta_I - \delta_S)t} = e^{i0} = 1$$

and $\quad e^{i(\phi_{\alpha\alpha}-\phi_{\beta\beta})} = e^{i0} = 1$

so that connectivity is well defined and given by

$$\int \beta_I^* \beta_S^* \, \underset{\sim}{O} \, \alpha_I \alpha_S \, d\tau$$ for all spinpairs in the sample

<u>or</u> system is perfectly <u>coherent</u>!

Thereafter, connectivity or coherent will evolve with time; evolution described by

$$e^{i2\pi(\underbrace{2\nu_{ref}}_{\text{2-quantum coherence}} \underbrace{-\delta_I - \delta_S}_{\text{Chemical shifts of } I \,\&\, S \text{ spins}})t}$$

Double-quantum coherence will evolve according to chemical shift between spins and ν_{ref}

Resultant phase difference accumulated after time t may be used to establish chemical shift between I & S spins connected by operator.

Similarly, for an operator that connects ϕ_3 & ϕ_2

$$\int \phi_3^* \, \underset{\sim}{O} \, \phi_2 \, d\tau$$
$$= \int \beta_I^* \alpha_S^* \, \underset{\sim}{O} \, \alpha_I \beta_S \, d\tau \cdot \left[e^{i2\pi(-\delta_I + \delta_S)t} \right] e^{-i(\phi_{\alpha\beta} - \phi_{\beta}}$$

so that if the system was perfectly coherent at $t=0$, e.g., following a $\frac{\pi}{2}$ pulse,
this (zero quantum) coherence will evolve with time as follows

$$e^{i 2\pi (\underbrace{0 \cdot \nu_{ref}}_{\boxed{\text{zero-quantum coherence}}} - \underbrace{\delta_I + \delta_S}_{\text{chemical shift difference between I \& S spins.}}) t}$$

and $\boxed{\text{any single-quantum coherence}}$ will evolve with time according to

$$e^{i2\pi(\nu_{ref} - \delta_I \pm J/2)t} \quad \text{(for coherence connected by } I_x \text{ or } I_y)$$

$$\text{or } e^{i2\pi(\nu_{ref} - \delta_S \pm J/2)t} \quad \text{(for coherence connected by } S_x \text{ or } S_y)$$

Resultant phase accumulated following duration t may be used to establish the spin-pair or spins connected by J-coupling.

Above provides the physical underpinning of multi-dimensional NMR.

Finally, note that double-quantum coherences evolve twice as fast as single-quantum coherences.

- Return to COSY Pulse Sequence & work out algebra for AX spin system

Pulse sequence : $RD - \left(\frac{\pi}{2}\right)_x - t_1 - \left(\frac{\pi}{2}\right)_x$ - acquisition

— At the beginning after RD:

$$\boxed{\sigma_0 = I_{1z} + I_{2z}}$$

— After the first 90° pulse ($\left(\frac{\pi}{2}\right)_x$):

according to rules, $I_z \xrightarrow{90^\circ_x} -I_y$

Therefore, $\boxed{\sigma_1 = -I_{1y} - I_{2y}}$

— During t_1-evolution period:

Both the chemical shifts and couplings will evolve.

(a) The effects of chemical shifts

according to rules, $I_y \xrightarrow{\Omega t I_z} I_y \cos\Omega t - I_x \sin\Omega t$

Therefore,

$$\boxed{\sigma_{2a} = -I_{1y}\cos\Omega_1 t_1 + I_{1x}\sin\Omega_1 t_1 - I_{2y}\cos\Omega_2 t_1 + I_{2x}\sin}$$

(b) The effects of scalar coupling:

according to rules, $I_{1x} \xrightarrow{\pi J_{12} t I_{1z} I_{2z}} I_{1x}\cos\pi J_{12}t + 2 I_{1y} I_{2z}\sin\pi J_{12}t$

and $I_{1y} \xrightarrow{\pi J_{12} t I_{1z} I_{2z}} I_{1y}\cos\pi J_{12}t - 2 I_{1x} I_{2z}\sin\pi J_{12}t$

Therefore

$$\sigma_{2ab} = -I_{1y}\cos\Omega_1 t_1 \cos\pi J_{12} t_1 + 2 I_{1x} I_{2z} \cos\Omega_1 t_1 \sin\pi J_{12} t_1$$
$$+ I_{1x} \sin\Omega_1 t_1 \cos\pi J_{12} t_1 + 2 I_{1y} I_{2z} \sin\Omega_1 t_1 \sin\pi J_{12} t_1$$
$$- I_{2y} \cos\Omega_2 t_1 \cos\pi J_{12} t_1 + 2 I_{2x} I_{1z} \cos\Omega_2 t_1 \sin\pi J_{12} t_1$$
$$+ I_{2x} \sin\Omega_2 t_1 \cos\pi J_{12} t + 2 I_{2y} I_{1z} \sin\Omega_2 t_1 \sin\pi J_{12} t_1$$

or

$$\boxed{\begin{aligned}\sigma_2 &= (-I_{1y}\cos\Omega_1 t_1 + I_{1x}\sin\Omega_1 t_1 - I_{2y}\cos\Omega_2 t_1 + I_{2x}\sin\Omega_2 t \\ &\quad \cdot \cos\pi J_{12} t_1 \\ &+ (2 I_{1x} I_{2z}\cos\Omega_1 t_1 + 2 I_{1y} I_{2z} \sin\Omega_1 t_1 + 2 I_{1z} I_{2x} \cos\Omega_2 t_1 \\ &\quad + 2 I_{2y} I_{1z} \sin\Omega_2 t_1) \cdot \sin\pi J_{12} t_1\end{aligned}}$$

— Following the second 90° pulse $((\frac{\pi}{2})_x)$:

according to the rules,

$$I_z \xrightarrow{90^\circ_x} -I_y$$
$$I_y \xrightarrow{90^\circ_x} +I_z$$
$$I_x \xrightarrow{90^\circ_x} I_x$$

Therefore,

$$\sigma_3 = (-I_{1z}\cos\Omega_1 t_1 + I_{1x}\sin\Omega_1 t_1 - I_{2z}\cos\Omega_2 t_1 + I_{2x}\sin\Omega_2 t_1)$$
$$\cdot \cos\pi J_{12} t_2$$
$$- (\cancel{+}2 I_{1x} I_{2y}\cos\Omega_1 t_1 + 2 I_{1z} I_{2y} \sin\Omega_1 t_1 + 2 I_{1y} I_{2x} \cos\Omega_2 t_1$$
$$+ 2 I_{2z} I_{1y} \sin\Omega_2 t_1) \cdot \sin\pi J_{12} t_2$$

Since I_z magnetization & product operators with both x, y magnetization cannot be observed,

$$\boxed{\begin{aligned}\sigma_3^{obs} &= \left(I_{1x}\sin\Omega_1 t_1 + I_{2x}\sin\Omega_2 t_1\right)\cdot\cos\pi J_{12} t_1 \\ &\quad -\left(2I_{1y}I_{2z}\sin\Omega_2 t_1 + 2I_{1z}I_{2y}\sin\Omega_1 t_1\right)\cdot\sin\pi J_{12} t_1\end{aligned}}$$

— 2 Final points

(1) Since $2\sin A\cos B = \sin(A+B) + \sin(A-B)$

and $2\sin A\sin B = \cos(A-B) - \cos(A+B)$

Can rewrite σ_3^{obs} as

$$I_{1x}\left[\tfrac{1}{2}\sin(\Omega_1+\pi J_{12})t_1 + \tfrac{1}{2}\sin(\Omega_1-\pi J_{12})t_1\right]$$

$$+\, I_{2x}\left[\tfrac{1}{2}\sin(\Omega_2+\pi J_{12})t_1 + \tfrac{1}{2}\sin(\Omega_2-\pi J_{12})t_1\right]$$

$$-\, 2I_{1y}I_{2z}\left[\tfrac{1}{2}\cos(\Omega_2-\pi J_{12})t_1 - \tfrac{1}{2}\cos(\Omega_2+\pi J_{12})t_1\right]$$

$$-\, 2I_{2y}I_{1z}\left[\tfrac{1}{2}\cos(\Omega_1-\pi J_{12})t_1 - \tfrac{1}{2}\cos(\Omega_1+\pi J_{12})t_1\right]$$

Therefore expect resonances at

$$\omega_1 = \Omega_1 \pm J_{12} \quad \& \quad \Omega_2 \pm J_{12}$$

after FT in the t_2 - dimension

(2) Evolution of I_{1x}, I_{2x}, $2I_{1y}I_{2z}$ and $2I_{1z}I_{2y}$ after second $\left(\frac{\pi}{2}\right)_x$ pulse determine FID in t_2 space. FT of evolutions of single-quantum coherences due to these operators give resonances at

$$\omega_2 = \Omega_1 \pm \pi J_{12}$$

$$\omega_2 = \Omega_2 \pm \pi J_{12}$$

<u>I_{1x}, I_{2x}</u> give the multiplet along the diagonal

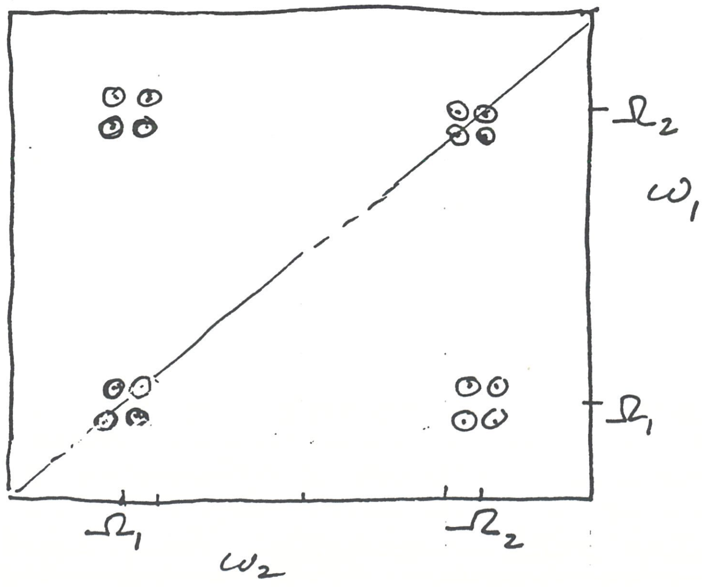
$$\omega_2 = \Omega_1 \pm \pi J_{12} \qquad \omega_1 = \Omega_1 \pm \pi J_{12}$$

$$\Omega_2 \pm \pi J_{12} \qquad \Omega_2 \pm \pi J_{12}$$

$2I_{1y}I_{2z}$, $2I_{2y}I_{1z}$ give the cross peaks

or $\quad \omega_2 = \Omega_1 \pm \pi J_{12} \qquad \omega_1 = \Omega_2 \pm J_{12}\pi$

$$\Omega_2 \pm \pi J_{12} \qquad \Omega_1 \pm \pi J_{12}$$



Multiple-quantum Filtration

Simplification of spectra by "largely" eliminating singlets in DQF-COSY, and AX and AB spin systems are largely eliminated in TQF-COSY.

DQF-COSY ≡ Double-quantum filtered-COSY

TQF-COSY ≡ Triple-quantum filtered-COSY.

In multiple-quantum filtration experiment, add an extra $\frac{\pi}{2}$ pulse immediately after end of the COSY sequence:

Pulse-sequence : $\left(\frac{\pi}{2}\right)_\phi - t_1 - \left(\frac{\pi}{2}\right)_\phi \left(\frac{\pi}{2}\right)_x - t_2$

and the third pulse follows instantaneously, and multiple-quantum coherence that happened to exist before the third pulse are converted back to observable magnetization.

Signals arising from different orders of multiple-quantum coherence can be separated out by suitable choice of phase cycling.

For example, double-quantum coherence is twice as sensitive to phase changes in its excitation sequence as is single-quantum coherence, double-quantum coherence evolves twice as fast as single-quantum coherence.

Therefore, if we shift the phase ϕ of the first two pulses (all pulses before the coherence is created) by $90°$, the phase of the detected signal which comes via the double-quantum coherence is inverted. Inverting the receiver phase (i.e., subtracting the $90°$ from the $0°$ experiment) then selects the component which passed through double-quantum coherence.

Double-quantum filtered (DQF) COSY experiment

$$\left(\frac{\pi}{2}\right)_\phi - t_1 - \left(\frac{\pi}{2}\right)_\phi - \Delta - \left(\frac{\pi}{2}\right)_x - t_2 \text{ (acquire)}$$

$\Delta \equiv$ very short phase switching delay

Full phase-cycling involves

$$\phi = x, y, -x, -y$$

and then $\phi = y, -x, -y, x$

with receiver $= +, -$ for alternate scans.

Analysis $\rightarrow$ Begin with $\phi = 0$ (x)

From earlier,

σ_3 (after second $\left(\frac{\pi}{2}\right)_x$ pulse),

$$= (-I_{1z}\cos\Omega_1 t_1 + I_{1x}\sin\Omega_1 t_1 - I_{2z}\cos\Omega_2 t_1 + I_{2x}\sin\Omega_2 t_1)$$

$$\cdot \cos\pi J_{12} t_1$$

$$+(2I_{1x}I_{2y}\cos\Omega_1 t_1 + 2I_{1y}I_{2x}\sin\Omega_1 t_1 + 2I_{1z}I_{2y}\cos\Omega_2 t_1 + 2I_{1y}I_{2z}\sin\Omega_2 t_1)\sin\pi J_{12}t_1$$

We can expand this result in terms of zero-quantum and double-quantum coherence components.

$$\sigma_3 = (-I_{1z}\cos\Omega_1 t_1 + I_{1x}\sin\Omega_1 t_1 - I_{2z}\cos\Omega_2 t_1 + I_{2x}\sin\Omega_2$$

$$\cdot \cos\pi J_{12}t_1$$

$$+\Big\{\tfrac{1}{2}[(2I_{1x}I_{2y} + 2I_{1y}I_{2x}) - (2I_{1y}I_{2x} - 2I_{1x}I_{2y})]\cos\Omega_1 t_1$$

$$+\tfrac{1}{2}[(2I_{1x}I_{2y} + 2I_{1y}I_{2x}) - (2I_{1y}I_{2x} - 2I_{1x}I_{2y})]\cos\Omega_2 t_1$$

$$+ 2I_{1z}I_{2y}\sin\Omega_1 t_1 + 2I_{1y}I_{2z}\sin\Omega_2 t_1\Big\}\sin\pi J_{12}t_1$$

During the phase cycle, all terms except those corresponding to double-quantum coherence cancel

$$(\sigma_3)_{\text{after phase cycling}} = \Big\{\tfrac{1}{2}(2I_{1x}I_{2y} + 2I_{1y}I_{2x})\cos\Omega_1 t_1 + \tfrac{1}{2}(2I_{1x}I_{2y} + 2I_{1y}I_{2x})\cos\Omega_2 t_1\Big\}\sin\pi J_{12}t_1$$

After the mixing pulse

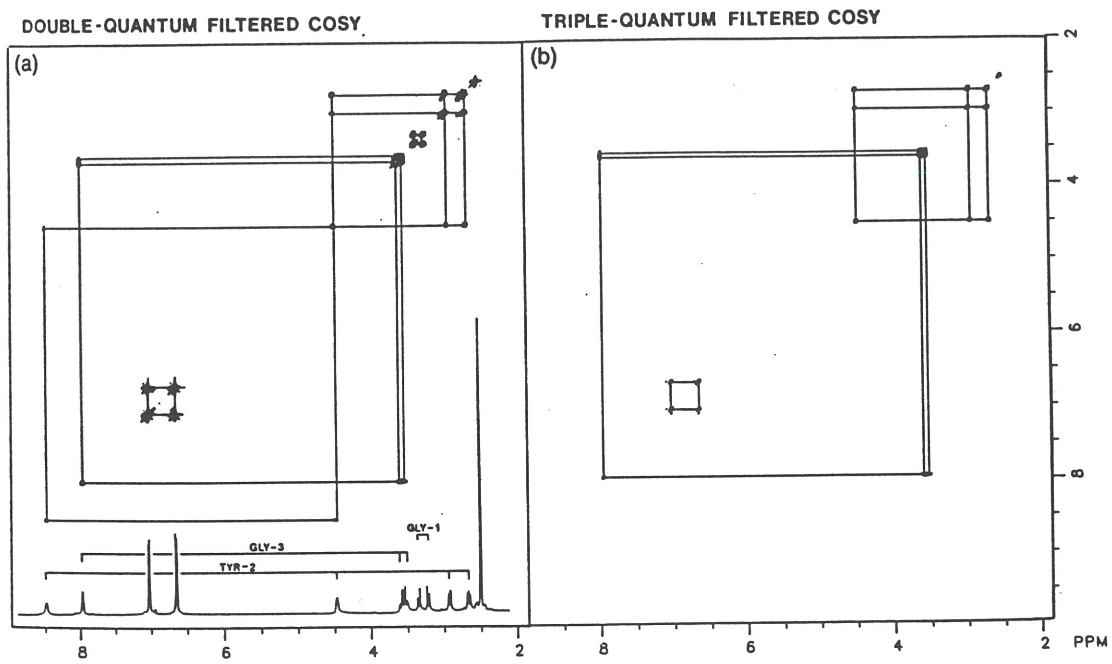
$$(\sigma_4) = \Big\{\tfrac{1}{2}(2I_{1x}I_{2z} + 2I_{1z}I_{2x})\cos\Omega_1 t_1 + \tfrac{1}{2}(2I_{1x}I_{2z} + 2I_{1z}I_{2x})\cos\Omega_2 t_1\Big\}\sin\pi J_{12}t_1$$

Each term evolves in t_2

$$\begin{aligned}
\sigma_5 = {} & \tfrac{1}{2}I_{1y} \sin \pi J_{12}t_2 \cos \Omega_1 t_1 \sin \pi J_{12}t_1 \cos \Omega_1 t_2 \\
& -\tfrac{1}{2}I_{1x} \sin \pi J_{12}t_2 \cos \Omega_1 t_1 \sin \pi J_{12}t_1 \sin \Omega_1 t_2 \\
& +\tfrac{1}{2}I_{2y} \sin \pi J_{12}t_2 \cos \Omega_1 t_1 \sin \pi J_{12}t_1 \cos \Omega_2 t_2 \\
& -\tfrac{1}{2}I_{2x} \sin \pi J_{12}t_2 \cos \Omega_1 t_1 \sin \pi J_{12}t_1 \sin \Omega_2 t_2 \\
& +\tfrac{1}{2}I_{1y} \sin \pi J_{12}t_2 \cos \Omega_1 t_1 \sin \pi J_{12}t_1 \cos \Omega_1 t_2 \\
& -\tfrac{1}{2}I_{1x} \sin \pi J_{12}t_2 \cos \Omega_1 t_1 \sin \pi J_{12}t_1 \sin \Omega_1 t_2 \\
& +\tfrac{1}{2}I_{2y} \sin \pi J_{12}t_2 \cos \Omega_1 t_1 \sin \pi J_{12}t_1 \cos \Omega_2 t_2 \\
& -\tfrac{1}{2}I_{2x} \sin \pi J_{12}t_2 \cos \Omega_1 t_1 \sin \pi J_{12}t_1 \sin \Omega_2 t_2
\end{aligned}$$

If the receiver is along the $+x$-axis, then the observed magnetization is

$$\begin{aligned}
\sigma_5^{\text{obs}} = {} & -\tfrac{1}{2}I_{1x}[\tfrac{1}{2}\sin(\pi J_{12}t_1 + \Omega_1 t_1) - \tfrac{1}{2}\sin(\Omega_1 t_1 - \pi J_{12}t_1)] \\
& \times [\tfrac{1}{2}\cos(\Omega_1 t_2 - \pi J_{12}t_2) - \tfrac{1}{2}\sin(\Omega_1 t_2 - \pi J_{12}t_2)] \\
& -\tfrac{1}{2}I_{2x}[\tfrac{1}{2}\sin(\pi J_{12}t_1 + \Omega_1 t_1) - \tfrac{1}{2}\sin(\Omega_1 t_1 - \pi J_{12}t_1)] \\
& \times [\tfrac{1}{2}\cos(\Omega_2 t_2 - \pi J_{12}t_2) - \tfrac{1}{2}\sin(\Omega_2 t_2 - \pi J_{12}t_2)] \\
& -\tfrac{1}{2}I_{1x}[\tfrac{1}{2}\sin(\pi J_{12}t_1 + \Omega_2 t_1) - \tfrac{1}{2}\sin(\Omega_1 t_1 - \pi J_{12}t_1)] \\
& \times [\tfrac{1}{2}\cos(\Omega_1 t_2 - \pi J_{12}t_2) - \tfrac{1}{2}\sin(\Omega_1 t_2 - \pi J_{12}t_2)] \\
& -\tfrac{1}{2}I_{2x}[\tfrac{1}{2}\sin(\pi J_{12}t_1 + \Omega_2 t_1) - \tfrac{1}{2}\sin(\Omega_2 t_1 - \pi J_{12}t_1)] \\
& \times [\tfrac{1}{2}\cos(\Omega_2 t_2 - \pi J_{12}t_2) - \tfrac{1}{2}\sin(\Omega_2 t_2 - \pi J_{12}t_2)]
\end{aligned}$$



Double- and triple-quantum filtered COSY spectra of the tripeptide Gly—Tyr—Gly. (Reprinted from ref. 7, Ch. 1 with permission.)